Localized Price Promotions as a Quality Signal in a Publicly Observable Network

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Abstract

This paper considers a quality signaling game in which consumers share information on quality with their neighbors in a social network. The firm chooses a set of consumers to whom to offer their product and a price across two stages of sales. When this publicly-observable localization is available, the cheapest way for the high quality seller to separate herself from a low quality seller is to initially offer the product to the smallest set of consumers who will saturate the network with word-of-mouth; targeting a set that generates less word-of-mouth will require more money-burning to successfully signal. The set is idiosyncratic and identifying it requires considering the connectivity of sets of nodes in the network collectively, rather than relying on individual measures of connectivity. Observable networks therefore enable credible signaling that is less socially wasteful than would otherwise be the case, and this benefit is greater the smaller is the set of consumers which will cover the network with word-of-mouth.

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1 Introduction

Post-purchase word-of-mouth influences consumer behavior (Godes and Mayzlin, 2004, 2009), and one way it does so is by helping a consumer learn about the quality of a product before she decides whether to buy it. This type of communication will benefit the seller of a high quality product and hurt the seller of low quality. An action by the seller that fosters communication—for example setting a low price (Kennedy, 1994) or restricting supply (Vettas, 1997)—is therefore more costly for a low quality product and so can be a signal of quality. However, as technology advances, the structure of communication networks is becoming more transparent. The graph of connections among Twitter users is partially public, and the sharing of information on a product in a social network is traceable. In May 2012 the staff of the social marketing platform RestEngine joined Twitter, promising to "help social app publishers send targeted one-to-one emails based on a subscriber's social graph... on a much larger scale at Twitter" (Grove, 2012). Novel services such as Klout Perks and Offergraph collect Facebook and Twitter profile information from consumers who sign up, and allow firms to make personalized promotional offers to consumers based on their position in the network of influence in a given product category. American Express, through its "Like, Link, Love" and "Sync" applications, has launched partnerships with Facebook and Foursquare to offer deals from third-party companies to consumers that are personalized according to the consumer's position in the relevant social graph.

Senders of information are therefore gaining an unprecedented ability to focus their activity in specific locations in a communication network. A firm's choice of where to use localized promotions within the communication network, and thus the amount of post-purchase communication to generate, can contribute to consumers' inference about the quality of their product. In this paper we consider the effect of the ability to publicly localize marketing efforts on the ability of a firm to signal the quality of its product to a population. We consider the example of fine targeting of specific nodes in a social graph, but the analysis can equally apply to any setting in which some degree of observable localization is possible.

While some targeting tools at a sender's disposal are new, the thread of consumer inference over a firm's coarse targeting behavior can be found in diverse settings. Movie audiences and movie producers know that critics' opinions are visible and influential, and so critics routinely infer that a movie not screened for them is bad. Conversely, opening a movie in select cities or on the festival circuit can serve as a signal of confidence in its quality, since press coverage and word-of-mouth from these early audiences will be available to the wider market. For a startup to aggressively headhunt a prominent worker can serve as a signal to the worker of the firm's credibility, since his prominence means that a poor firm will suffer a large reputation loss if he reports to his connections the true quality of the firm. A firm's decision to use a narrow launch for a new consumer good can signal to those offered the good during the launch that the product is good, since a good product will benefit from communication between early adopters and later customers, while a bad product

will suffer.

In the model we present below, a firm has a product of exogenous quality that is either high or low; consumers value a high quality product, but would never buy a product they knew to be of low quality, and cannot verify quality before purchase. Consumers will consume the product only once and so the firm cannot rely on repeat purchase to recoup the cost of signaling. The firm chooses a launch strategy that defines in each of two periods, 'today' and 'tomorrow', a subset of consumers to whom the product will be made available and at what price. Consumers observe these choices and choose whether or not to buy the product, given their belief on quality, which can come either from Bayesian updating over the firm's strategy or, tomorrow, direct observation of the experience of a network neighbor who purchased today.

When the location of the firm's first-round offers is publicly observable, the cheapest way to signal quality is to target the smallest set of consumers that will saturate the network with word-of-mouth (a *minimum dominating set* of the network graph[1]) and offer them a price discount (equivalently, take a large enough money-burning action) to convince them to buy today. Identifying such a set requires considering the connectivity of sets of nodes taken together, rather than relying on individual measures of connectivity. By visibly localizing offers today to a set of consumers strategically chosen for their ability to generate network-covering communication about quality, the firm signals to these consumers that it is willing to restrict activity today, with both parties knowing that if the quality of the product is bad, such a strategy results in no business tomorrow. This set is targeted in the launch stage in all equilibria that feature least-cost separation of types, and is characterized by dispersed, non-concentrated targeting, in order to avoid wasteful redundancy in generated word-of-mouth. This is consistent with the argument in Godes and Mayzlin (2004) that "more dispersed buzz may be better than concentrated buzz". Dispersion of launch locations across the network here is a stronger signal of high quality than a concentrated launch, since dispersion is associated with a greater volume of the irredundant word-of-mouth that benefits good products over bad.

Since the targeted set is the most parsimonious that will saturate the network with word-of-mouth, separation in these equilibria is driven entirely by the communication generated by targeting: only those consumers targeted in the first period must infer quality from the firm's strategy, with all others learning it by word-of-mouth before purchasing. For these targeted consumers to be convinced to buy requires a costly signaling action, captured in the model as a price discount.[2] Targeting a minimum dominating set is the cheapest signal because to separate while targeting a different set requires a greater amount of directly costly signaling—a greater price discount—to generate inference of high quality by those consumers who are informed directly or not covered by word-of-mouth. In this way a judiciously targeted launch reduces the amount of directly costly

[1]This is the lowest-cardinality set such that all nodes in the graph are either in the set or one degree away from a node in the set.

[2]This price discount is equivalent to any "money-burning" signaling action, such as non-informative advertising expenditure.

signaling that the high quality firm must undertake in order to separate. This means that public observability of a communication network—when it is exploited strategically—is valuable to both the firm and society since it enables credible signals of quality to be less wasteful than would otherwise be the case. Further, since adding links to a network never increases the size of the set targeted in separating equilibria, marginal links have weakly positive value to the seller of a high quality product.

The magnitude of the costly 'money-burning' action required to convince the first-targeted to buy the product depends also on consumer beliefs, in a way familiar from traditional signaling games. While all separating equilibria see the aforementioned set of consumers targeted first, there exist two classes of separating equilibria. In the first, the high and low quality firms are separated immediately. This happens by the firm engaging in sufficient price discounting at launch that a low quality firm would be unprofitable even if all targeted consumers bought the product; by a standard Intuitive Criterion argument on consumer beliefs, this rules out for the targeted consumers the possibility that the product is of low quality and so ensures that they will be willing to purchase.

In the second class of separating equilibria, the two types of firms at first pool and are separated as word-of-mouth transmits quality information. In this case the firm engages in a smaller amount of price discounting that is consistent with a consumer being just willing to buy a product of the average quality in the distribution. Since this would be profitable for the seller of either a high or low quality product, this case requires consumers' beliefs on quality not to be too pessimistic. In such an equilibrium, high quality products are "discovered" in the market rather than being separated right away. This is more profitable for the high quality firm since it requires less costly signaling, and in Theorem 2 below we show that under the *passive conjectures* refinement (Rubinstein, 1985, Rasmusen, 2001) on consumers' beliefs this is the unique equilibrium in the model. We go on to show that the strategies for a high quality firm in both classes of signaling equilibria are robust to relaxing the assumption that consumers observe the structure of the network.

The strategies for the firm in both classes of separating equilibria are consistent with the growing interest in tailoring offers to consumers based on their position in online social graphs. Many prominent brands, including Microsoft, Samsung, Sony, Westin Hotels, Virgin America, Revlon, Red Bull, and Whole Foods Market have used Klout Perks and American Express's "Like, Link, Love" and "Sync" programs to offer products either free or at a discount to consumers based on their network position. Our model suggests that targeting offers to consumers chosen for their visibility to other consumers has a twofold benefit for the firm: the strategy that maximizes the spread of word-of-mouth about the product is also the most efficient way to signal the quality of the product to the targeted consumers. These strategies based on online social networks are similar to older, more established strategies of firms attempting to discover by survey and sociometric analysis the networks of relationships among a small set of potential customers, in order to decide where to direct costly sales effort. The use of this approach by pharmaceutical firms to identify the network of influence among physicians in drug prescription behavior has been well documented in

a large literature dating at least to Coleman et al. (1966) and reviewed in Iyengar et al. (2011b). An implication of the model is that firms seeking to employ this kind of strategy should consider the connectivity of sets of consumers in the network when deciding who should receive the initial promotions, rather than simply considering the connectivity of individual consumers. For more general interpretations of nodes in the network, strategies of initially launching a product in select markets—for example the common practice of opening niche movies in New York and Los Angeles weeks before their wide release—or giving a product away to public figures are also consistent with the separating equilibria in the model.

This paper contributes to a growing literature on the relationship between firm strategies and the shape of the communication network among consumers. While we focus on the role of the firm's launch strategy in driving inference about quality, there are many reasons why the shape of the network matters. One is awareness: the best way to disseminate information will depend on how it will be retransmitted among the population. This case has been studied in, for example, Galeotti and Goyal (2012), Galeotti and Goyal (2009), Zubcsek and Sarvary (2011) and Campbell (2012), where, in general, knowledge of the communication network among the population is shown to be valuable for the informer. A second motivation is persuasion: the value an agent derives from some product or action can depend on its valuation by his network neighbors, and so for the informer convincing one consumer can have recursive effects. This setting corresponds to graphical games as analyzed in Kearns et al. (2001), Galeotti et al. (2005) and Jackson and Yariv (2008). In the model we present in this paper, we abstract from the informing and persuading roles of communication networks; consumers know that the product exists, and the values of 'good' or 'bad' versions of it are commonly held and fixed. This means that the model will isolate the signaling role and how it depends on the architecture of the communication network.

The transmission of quality information across a network features also in Navarro (2006), which models a consumer network in which quality information is transmitted across links. A firm chooses quality and price, and consumers simultaneously decide whether or not to buy, where willingness-to-pay is in equilibrium consistent with the information they will receive as a function of other players' decisions. Godes and Mayzlin (2009) considers a firm's decision to announce a reference program that commits it to facilitating information flows from early adopters to potential late adopters. Most relevant to the current model is the case in which consumers are uncertain about product quality, in which case committing to a reference program can function as a signal of quality. Liu and Schiraldi (2012) models an environment in which a decision-maker with a product of quality unknown to any party chooses a number of sales offers to make in each of N periods. Potential adopters observe how many offers have previously been made and how many people previously adopted and receive some noisy private information about the product's quality. The problem for the decision-maker is whether to seek to foster an information cascade across the population of potential adopters. By contrast, the model below admits consumers both to infer quality from a firm's launch strategy and learn quality directly from previous adopters by

word-of-mouth.

The model below also builds on the literature on communication in general as a factor in quality signaling. Bagwell and Riordan (1991) models a setting in which some consumers are initially informed about quality and some are initially uninformed. This, with a production cost differential between high and low quality products, admits that an initially high price can be a signal of product quality. Kennedy (1994) models the quality signaling problem for a firm that can set price, absent such a cost differential that drives the familiar single-crossing property (following Spence, 1973), and demonstrates that a low and rising price can be a signal of quality due to the word-of-mouth that will benefit a good product and hurt a bad one. Similarly, Vettas (1997) demonstrates that restricting quantity supplied can act as a signal of quality when information on product quality can spread by word-of-mouth. The present paper builds on Bagwell by putting observable structure on the spread of information from the informed to uninformed, and follows Kennedy and Vettas in assuming no production cost differential. In this framework the location of initial targeting reduces the signaling burden that must be borne by price and volume: the easier it is to saturate the network with observable word-of-mouth, the less money-burning or supply restriction is required to signal high quality and thus kick-start purchasing by consumers. Finally, the notion that location in general can be a signal of quality is explored in Vettas (1999), which demonstrates that location in product space can be a signal of quality.

The framework of the present model incorporates a network graph as, first, a model for the transmission of information about quality during a signaling game, and, second, as the strategy space of the firm's launch location decision. A natural constraint on the literal interpretation of the model is that a network graph can in practice be highly complex. A rich literature in computer science has explored problems of choosing nodes in a graph to maximize the recursive spread of influence through the network (for example Domingos and Richardson, 2001, Richardson and Domingos, 2002, Kempe et al., 2003). These graph coverage problems are known to be NP-hard (Garey and Johnson, 1979). The literal application of the model presented below is therefore restricted to settings in which a relevant network is 'small enough' to be understandable to the decision-maker. This could mean a small number of individuals, or, if we interpret a 'consumer' more liberally (for example as a cluster of consumers, a geographic area or a firm), a small number of relevant decision units on the consumer side. More generally, the problem of translating the model to settings with less well-understandable networks is discussed below following the analysis. Strategies in the spirit of the focal set in the separating equilibria of the model can be identified even in situations without a network that is literally observable at the finest level.

The paper proceeds as follows. Section 2 presents the model. Section 3 identifies the equilibria of the game, and its Theorem 1 identifies those that survive the application of the Intuitive Criterion. Section 4 and its Theorem 2 identify equilibria surviving the application of the passive conjectures refinement. Sections 5 considers the assumption that consumers observe the network structure and the implications of relaxing it, and 6 considers the applicability and implications of

the assumptions on how information is shared by consumers. Section 7 concludes.

2 Model

Consider a dynamic game with incomplete information. A firm will encounter a set of consumers arranged in a social network[3], and has two periods in which to make sales offers to consumers. Each time the firm can choose a price and a subset of consumers to whom the price will be offered, and the consumers who receive a price offer can choose to accept or reject it. Between the two rounds, communication will reveal details of round 1 transactions to neighbors of those consumers who transacted. The remainder of this section will formalize this game.

2.0.1 Players

There is a set $C = \{1, ..., n\}$ of risk-neutral consumers. Each consumer is a node in an undirected and connected graph (C, g)[4], where g is a real-valued $n \times n$ matrix in which g_{ij} represents the relationship between consumers i and j: $g_{ij} = 1$ if there is an edge joining i and j and 0 otherwise. $g_{ij} = g_{ji}$ since the graph is undirected; let $g_{ii} = 0$. Consumer j is considered a "neighbor" to consumer i if $g_{ij} = 1$. Since C is assumed fixed, for convenience denote the graph by g. Let $\Gamma \subseteq C$ be some set of consumers, and let $\gamma = |\Gamma|$ be its cardinality.

The open neighborhood of consumer i is $N(i) = \{j : g_{ij} = 1\}$. The open neighborhood of Γ is $N(\Gamma) = \cup_{i \in \Gamma} N(i)$. Let $\Omega(\Gamma) = N(\Gamma) - \Gamma$; this is the set of consumers in the neighborhood of Γ who are not themselves in Γ. For notational consistency denote $\Omega(i) \equiv N(i)$. In turn let $\omega(i) = |\Omega(i)|$ denote the number of neighbors to consumer i (equivalently i's degree) and $\omega(\Gamma) = |\Omega(\Gamma)|$.

A dominating set of the network graph is some Γ such that $\Gamma \cup \Omega(\Gamma) = C$. Denote by Γ^* a **minimum dominating set**, which is a dominating set with the lowest possible cardinality. The **domination number**, denoted $\gamma^* = |\Gamma^*|$, is the cardinality of a minimum dominating set; it is the smallest number of consumers such that marking γ^* would result in every single consumer either being marked or having a marked neighbor. While γ^* is unique for a given network, Γ^* is not: there can in general be more than one minimum dominating set in a given graph. Note that since g is connected, the domination number cannot exceed $\frac{1}{2}n$ (established in Ore, 1962), although it will in general be smaller.

The example in Figure 1 illustrates these concepts. This 'double star' network has 8 consumers in total, a domination number of 2, and an associated minimum dominating set consisting of the shaded central consumers: each consumer is either shaded or a direct neighbor of a shaded

[3]While we focus on the application of a social network, depending on the setting we could also apply the model to any type of network structure that admits localized strategies, for example geographic, professional or collaboration networks.

[4]That the graph is connected is without loss of generality, since an unconnected graph would in this model be qualitatively identical to each of its connected components.

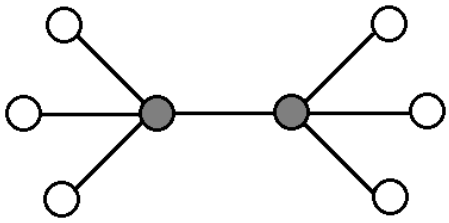

Figure 1: Double star network with minimum dominating set shaded

consumer. This configuration is an example of a network with a unique minimum dominating set.

There is one firm which produces a good for which consumers have unit demand.[5] The good is of (exogenous) quality $q \in \{q_L, q_H\}$, where q_H is realized with probability α. The firm knows the quality of its good. Consumers are familiar with the distribution of quality but cannot directly observe the quality of good provided by the firm. Call the expected quality $\bar{q} \equiv \alpha q_H + (1-\alpha)q_L$, where $q_L < 0 < \bar{q} < q_H$. Production costs are zero for both types of firm. This could be considered a normalization of production costs, or more literally that the goods to be sold have already been produced and the problem for the firm in this model is simply to sell them. This guarantees that either type of firm is always willing to sell at any positive price if possible and that a traditional signaling argument based on production cost differential is ruled out in order to isolate signaling via launch strategy. Denote the type of a firm with a high quality product H and the type of a firm with a low quality product L. It is not possible for the firm and consumer to write contingent contracts, such as warranties, legal recourse or money back guarantees.

The consumers' network g is observable by the firm. The firm's knowledge of the whole network best approximates settings in which information on the network is cheap to acquire, either because the relevant population is small or because technology to analyze the network is effective. An example of such a case is found in the study of opinion leadership and social contagion of Iyengar et al. (2011b), in which the authors employ survey techniques and analysis to construct the relevant network of influence among physicians in three large U.S. markets. The range of settings covered by the assumption of a network observable to the firm is likely to increase, for example as technology develops and the sophistication of online interactions continues to grow.[6] Narayanan and Shmatikov (2009) note that "[a] commercial enterprise, especially one specializing in behavioral ad targeting, can easily obtain an anonymized social-network graph from the network operator for advertising purposes."

[5]This is consistent with the good being a durable or an experiential product, and rules out the well-understood signaling mechanism of repeat purchase in order to isolate the effect of communication among consumers.

[6]The technical burden borne by this assumption also differs in how we interpret 'consumers' in the model. For example, we could view 'consumers' as audiences of distinct websites, and the links between them as hyperlinks or cross-posts.

We assume that the network g is also fully observed by consumers. This is perhaps more likely to be the case in networks that are small in the sense of having fewer consumers, or in settings where the network structure is sufficiently simple to be easily understandable, such as a network graph composed of distinct cliques who are completely connected within the clique but have few connections outside. In Section 5 below, we consider the implications of relaxing the assumption that consumers observe the network graph. With some additional structure, the strategies and outcomes we identify for the high type firm in equilibrium in the case when consumers can observe the network are preserved when they cannot, but the low type firm may do better when consumers cannot observe the network, since this makes it harder for consumers to make inference from the firm's behavior.

2.0.2 Moves

Let $t = 1, 2$ index the rounds. The structure of play is as follows:

0 Nature chooses $q \in \{q_L, q_H\}$.

1 a The firm chooses price p_1 and the set Γ_1 of consumers to whom the price will be offered.

b Sales round 1: consumers observe p_1 and Γ_1. If consumer i was offered a price, she chooses to buy ($b = 1$) or not buy ($b = 0$).

c If consumer i accepts the sale, consumer i and each neighbor of consumer i observe the quality of the good.

2 a The firm chooses price p_2 and the set Γ_2 of consumers to whom the price will be offered.

b Sales round 2: consumers observe p_2 and Γ_2. If consumer i was offered a price and has not bought yet, she chooses to buy ($b = 1$) or not buy ($b = 0$).

The strategic variables for the firm are therefore the two price and visitation policies, making a strategy for the firm as follows: $\sigma_F = \{p_1, p_2, \Gamma_1, \Gamma_2\}$, where p_t specifies the price at time t and Γ_t the set of consumers to whom the product is offered at time t. The stage 2 strategy for the firm can depend on history (the firm's own stage 1 offer and the profile of consumer responses).

The firm can choose a policy that can vary price according to round and can specify the set of consumers to which each round's price will be offered. The price and set chosen by the firm in each round is fully observable to consumers in that round (again in Section 5 we relax this observability assumption). Stage 1c is the "communication" stage, in which any purchases made in stage 1 are observed by the neighbors of the consumer who made the purchase. This reveals to the neighbor with certainty the quality of the good. Note that Γ_2 is not restricted to be disjoint with Γ_1.

The structure of the sharing of information on quality has, for tractability, several important restrictions. First, it occurs non-strategically, with consumers not weighing costs and benefits of sharing information. Second, it is fully honest, and so we do not allow the possibility that

consumers may seek to intentionally mislead neighbors about the quality of a product. Third, information is shared precisely, and so we rule out different tastes for the product. Taken literally, these restrictions capture settings in which tastes for the product are relatively homogeneous, consumers have an intrinsic desire to provide their neighbors with accurate recommendations, and the costs of sharing information are small relative to the benefit that a consumer derives from giving useful information to neighbors. Following the analysis of the model, in Section 6 we will consider some implications of these assumptions and in what settings they may be most appropriate.

A strategy for consumer i who is offered a price in stage 1 is $\sigma_{1,i} = \{b|p_1, \Gamma_1\}$. A strategy for consumer i who is offered a price in stage 2 also incorporates information I gained in stage 1: $\sigma_{2,i} = \{b|p_1, \Gamma_1, p_2, \Gamma_2, I\}$. In round 2, denote by $\mathcal{I}$ the set of 'informed' consumers: those who have a neighbor who purchased in round 1 and so know for sure in round 2 the quality of the product. Call the belief held by household i on the firm's quality at some time $\hat{q}$.

2.0.3 Payoffs

The ex ante payoff to a consumer i who buys at any point during the game at a price p is

$$E(U_i) = \hat{q} - p, \tag{1}$$

where $\hat{q}$ is the belief held by the consumer at the time of purchase, which may have been updated from their prior held at the start of the game. A consumer who never buys receives a payoff of zero. Each consumer thus follows a simple purchasing rule: they will always be willing and able to purchase one unit of good when the price does not exceed expected quality.

The payoff to a firm of type j is as follows, where the y_t is the number of buyers from the firm in round t and p_t the price set by the firm in round t:

$$\pi_j = y_1 p_1 + y_2 p_2 \tag{2}$$

Note that the game differs slightly from a graphical game or a network game, as defined in Jackson and Yariv (2008); here, a consumer's location in the network determines what information she sees, but neighbors' actions do not *directly* affect the payoff earned by a consumer.

3 Equilibria

The object of interest will be perfect Bayesian equilibria that survive the application of the Intuitive Criterion (Cho and Kreps, 1987). The components of an equilibrium are strategies for each type of firm $\sigma_{F,j} = \{p_1, p_2, \Gamma_1, \Gamma_2\}$, $j = H, L$, strategies for each consumer $\sigma_{1,i} = \{b|p_1, \Gamma_1\}$, $\sigma_{2,i} = \{b|p_1, \Gamma_1, p_2, \Gamma_2, I\}$ and supporting beliefs.

This section will describe the equilibria in this game by class, and then apply the Intuitive Criterion. There are three broad classes:

Round 1	Round 2
Separated	Separated
Pooled	Pooled
Pooled	Separated

First, the case in which the two types are separated before the first sales round; second, the case in which the two types pool and both types make sales in both periods; third, the case in which the types pool in the first round and are separated by word-of-mouth between rounds.

Of these, the third class is novel, being that class in which separation takes place entirely through communication that is fostered by the firm's targeting behavior and not through any costly signaling action. In this case no consumer can infer the firm's true type before buying. By contrast, the other two classes of equilibria are analytically similar to pooling and separating equilibria in a traditional signaling game. In general all three types of equilibria exist and survive the Intuitive Criterion (Theorem 1 below). The most profitable equilibria for the type of firm with a high quality product belong to the third class. In Section 4 we show that a further refinement on consumers' beliefs, the passive conjectures refinement, rules out the other two classes of equilibria.

3.1 Equilibria surviving application of the Intuitive Criterion

3.1.1 Separation before round 1

There exist equilibria in this game analogous to separating equilibria in one-stage signaling games: the high type firm takes an action to distinguish itself from the low type firm before any consumers have purchased. In these equilibria, the high type firm makes sales in both rounds and the low type makes no sales in either round.

Definition 1. *An* S1 equilibrium *features separation before round 1, so that the high type firm makes sales in both rounds and the low type firm makes sales in neither round, and no sale takes place in either round to a consumer who does not know the true type of the firm.*

In an S1 equilibrium high types play a strategy that is sufficient to preclude profitable imitation by low types in either round. All consumers can infer from this strategy that the firm is of high type, even before anyone has consumed the product and information on its quality spreads. In this class of equilibria an argument analogous to that in Cho and Kreps (1987) applies: a qualitatively unique S1 equilibrium survives the application of the Intuitive Criterion. Here this will be the S1 equilibrium with the lowest cost to achieve separation, the analog of the "Riley outcome" in a standard signaling model. The following result characterizes the precise nature of the surviving S1 equilibria:

Lemma 1. *In all S1 equilibria surviving the Intuitive Criterion, the high type firm offers a price of zero to consumers in a minimum dominating set of the network graph in round 1, and offers a price equal to q_H to all other consumers in round 2. The high type firm earns a payoff of*

$$\pi_{H,S1} = (n - \gamma^*)q_H \tag{3}$$

The proof of this result, along with those of all others to follow, appears in the appendix. In the surviving S1 equilibria, the high type firm offers the product at a price of zero in round 1 to the lowest-cardinality set of consumers whose consumption experience will reveal to all others the true quality of the good. In Figure 2 this is the set of shaded nodes. This choice by the high type firm is a credible signal of quality because it imposes costs on the low type firm. By imitating this strategy, the low type firm would earn no revenue in the first round (because of the zero price) and no revenue in the second round (because all remaining consumers will learn the true quality of the product by word of mouth). Among the strategies for the high type firm which impose these costs, offering the zero price to the minimum dominating set is the most profitable since it leaves the most possible remaining consumers to be served at the high price q_H in round 2.

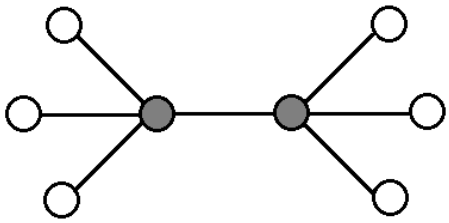

Figure 2: Minimum dominating set targeted in first round by high type in S1 equilibrium

All remaining consumers learn the true quality of the product by word-of-mouth, and so in round 2 the high type offers the product to all remaining consumers at a price of q_H, precisely willingness to pay for the high quality product. Since we have seen that the low type firm cannot profitably imitate this strategy—price is zero in round 1, and by round 2 all consumers have learned quality—and so by the Intuitive Criterion the belief of the consumers targeted in round 1 is that the product is certainly of high quality, making them willing to accept the product.

With this strategy, the firm sells at zero price to all the shaded consumers. Why, then, is the surviving S1 strategy superior for the high type firm than, say, offering a sufficiently negative price to, say, just *one* of the shaded consumers? Generating such inference requires a weakly more costly price discount than it pays back to the high type in the second round[7]. In Figure 2 the latter strategy would require a discount sufficient to convince three of the outside consumers (who

[7]The price discount required is strictly more costly unless some consumer in the minimum dominating set has only a single neighbor outside the set.

would not learn quality through communication) that the firm was of high type. Inference is more costly when not backed by word-of-mouth. This means that while there are separating equilibria in which the firm generates targets a smaller number of consumers than the number that blanket the network with word of mouth information on quality, these are always inferior for the high type firm than the separating equilibria that targeted promotions to the minimum dominating set in the first round. As we discuss in 5 below, this feature of the efficient S1 equilibria, with some mild additional assumptions, makes the associated high type strategy robust to relaxing the assumption that consumers observe the network structure.

The strategy for the high type firm in the efficient S1 equilibrium is consistent with a examples in which a firm seeks to give away a product, or sell it at cost, to a set of consumers chosen for their collective visibility to the rest of the network of potential consumers. Services that allow firms to make special giveaways to consumers selected by network position, such as Klout Perks and Like, Link, Love by American Express, have this feature. Both well-established and new brands offer free products through these platforms; typical examples of giveaways from Klout Perks[8] include new product launches from new firms and familiar brands such as Dove, Lays and Samsung, and free preview screenings of new movies and TV shows. The signaling motivation for this strategy from the S1 equilibrium is that the targeted consumers are willing to accept the free product, making inference that if the product were bad, their influence in the relevant network would be destructive, and other consumers will be able to either make the same inference from this targeting behavior or learn the product's quality through the resulting word-of-mouth.

Giving products away to celebrities can also have a motivation similar to the S1 strategy. A recent example of this approach in action is the launch of "Beats by Dre" headphones. The product was given away to musicians and athletes, both in versions available to consumers, and in versions specially designed for the individual, with the result that "the company's headphones have shown up in countless music videos, including Lady Gaga's hit 'Poker Face', and in NBA locker rooms, including around LeBron James's neck" (Sanburn, 2013). This strategy raised mild controversy at the London 2012 Olympics, as they "skirted strict rules on ambush marketing by sending British team members special versions of the Beats range branded with union flag colours" (Sweney, 2012). Again, as well as helping to raise awareness of the product and influence consumer tastes, the targeted giveaway and the expense of the personalized designs can help to signal the quality of the product.

All else being equal, a network whose minimum dominating set has smaller cardinality is associated with cheaper signaling in S1 equilibria, and so in a world in which targeting is possible, a network with a smaller minimum dominating set requires less seeding to saturate with word-of-mouth and so requires less money-burning. We can formalize this notion by showing that adding links to a given network weakly increases the high type firm's payoff in an S1 equilibrium:

[8] http://klout.com/perks

Corollary 1. *In an S1 equilibrium, the marginal payoff to the high type firm of adding a link to the network is either* 0 *or* q_H*, depending on the location of the link.*

This follows directly from the fact that the domination number never increases when an edge is added. An increased proliferation of communication channels in a network of fixed size must therefore weakly reduce the amount of money burning required to signal type.

In this class of equilibria we need not take literally that the signaling action is a price discount; it would be precisely equivalent to have a high price in both rounds and combine the targeted set with a 'money-burning' action large enough to convince the targeted set to buy. For example, in a setting in which advertising expense or time-consuming sales effort (as in the Coleman et al. (1966) pharmaceutical marketing study) are the money burning mechanism, it remains true that the magnitude of money burning required to signal type decreases as the breadth of launch required to generate network-spanning communication decreases. On the other hand, if we do take literally the notion of a price discount, the incentive to signal and to generate communication can be a motivation for launch-stage penetration pricing that is different to the motivation in, for example, Katz and Shapiro (1986) of penetration pricing to encourage adoption of a product with network externalities in consumption.

Finally, Lemma 1 also guarantees a lower bound on the payoff of the high type under the Intuitive Criterion, which in turn guarantees that no equilibrium in which no sales are made survives the Intuitive Criterion.

Lemma 2. *No equilibrium in which the high type receives a payoff lower than* $\pi_H = (n - \gamma^*)q_H$ *survives the application of the Intuitive Criterion.*

The S1 strategy is a profitable deviation for the high type firm from any hypothetical equilibrium that gives a lower payoff; under any consumer beliefs satisfying the Intuitive Criterion such a deviation will always be accepted by consumers.

3.1.2 No separation

Next, consider the cases in which both types of firm make sales in both rounds, so that there is no separation of types except by word-of-mouth. This can only be equilibrium play either if a 'pooling' price—one which does not exceed average quality—is set by both types in both rounds, or if there are no sales in the second round. In either case this means that in these equilibria the high type does not attempt to exploit its distinction from a low type, even though neighbors of consumers who buy from the high type in round 1 learn of its type before round 2, and so a 'pooled price' prevails in both rounds:

Definition 2. *An equilibrium with* ***no separation*** *(NS) features each type of firm set a price no greater than a consumer's willingness to pay for a product of the true expected quality in the distribution.*

We can quickly demonstrate the following result:

Lemma 3. *No equilibrium in which no offers are made in the second sales round survives the application of the Intuitive Criterion.*

In the surviving NS equilibria, both types of firm offer some price to a given group in round 2, and some price to a given nonempty group in round 2. The high type firm makes no attempt to exploit the revelation of information about its type, and the two types pool and make sales in both rounds. For an equilibrium of this type to exist, it must be that the high type firm prefers to make sales at a pooled price the set targeted in the second round rather than to make sales at a higher, separated price to informed neighbors of the consumers targeted in the first round. It follows that no NS equilibrium can exist in which a dominating set is targeted in the first round.

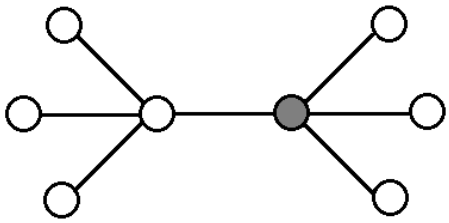

Figure 3: Can exist NS equilibrium with only the shaded consumer targeted in first round

An example in the double star network is in Figure 3. If consumers believe that the firm is of low type if they observe more than one consumer targeted in round 1, and if the high type firm prefers to sell to the seven unshaded consumers at the pooled price $\bar{q}$ than to sell to the four neighbors of the shaded consumer at the separated price q_H, then a NS equilibrium exists with the shaded consumer targeted in round 1, all others targeted tomorrow and price equal to the pooled price $\bar{q}$ in both rounds.

For an equilibrium of this type to survive the application of the Intuitive Criterion, there must be no deviation that benefits only the high type firm. By Lemma 2, this immediately rules out NS equilibria that are inferior for the high type firm to a surviving S1 equilibrium. It also must be the case that to expand the targeted set in the first round at a pooled price must be beneficial to either type of firm. Continue to consider the proposed NS equilibrium in the network of Figure 3. If consumers would accept it, the high type firm will benefit from expanding the first round targets to the minimum dominating set: it will be able to sell to all other consumers at separated price q_H tomorrow. But this may also be beneficial for the low type firm: it will make no sales tomorrow, as all will have learned its type, but will make an extra sale today at the pooled price $\bar{q}$. This example demonstrates that the Intuitive Criterion cannot in general rule out consumers' 'pessimistic' beliefs on the breadth of the firm's launch strategy, since expanding the set of first round offers at positive price can be beneficial for either type of firm.

3.1.3 Separation between rounds

In this class, the two firms pool in round 1 and separate in round 2. High type firms sell in both rounds, and low type firms sell only in round 1.[9]

Definition 3. *An* S2 equilibrium *features separation between rounds, so that there is a common, pooled price in round 1 for both types of firm, and a separating, high price in round 2 for the high type firm.*

In round 2 the high type firm can offer a price of q_H to neighbors of the consumers who bought in round 1; that is, those consumers who learn the firm's type during the communication stage. This strategy is also that to which a high type firm could deviate to circumvent the NS equilibria, should it wish to do so. In the double star example from the preceding section, this deviation would constitute the high type firm playing $\Gamma_2 = \Omega(\Gamma_1)$ and $p_2 = q_H$.

This communication-driven class of equilibria helps to dictate an upper bound on the number of sales in round 1 in any equilibrium that survives the application of the Intuitive Criterion:

Lemma 4. *In equilibria surviving the Intuitive Criterion, the number of consumers targeted in round 1 never exceeds the domination number of the network graph.*

The proof again appears in the appendix. Note that in a deviation by the high-type firm away from a hypothetical equilibrium with a larger number of targeted consumers, the location of offers is important: by locating offers at the minimum dominating set, consumers can be assured that, if all of those offers are accepted, word-of-mouth will reveal true type to the remainder of consumers. It is this feature that guarantees that such a deviation would not be profitable for the low type firm. As discussed in Section 2, the domination number of a connected graph never exceeds one half of its nodes, but is in general lower and depends on the precise architecture of the network.

This result seems to place a very loose upper bound on first round sales; this is a relic of the fact that the model is, for tractability, restricted to have two rounds. The more general implication of Lemma 4 is that when a targeted launch is possible, the ability to strategically locate early sales to spread information about quality pushes strongly towards a launch whose breadth is capped at that needed to spread the word in a suitably 'short' time.

This result demonstrates that in such settings the number of sales made in the first round of the game is bounded above in a way that depends on the structure of the communication network. Because the firm can target sales it can visit an appropriately small set of consumers to exploit local communication, which is profitable only for the high-type firm, being the one not hurt by revelation of true quality.

[9]Note that although the firms are 'separated' by word-of-mouth and experience different round 2 outcomes, in equilibria of this class the two types of firm may choose identical strategies in round 2, since the low type firm will be indifferent between mimicking the high type strategy (thus making no sales) and setting some different strategy.

3.2 Surviving equilibria

This result also completes the characterization of equilibria surviving the Intuitive Criterion.

Theorem 1. *The set of surviving equilibria consists of:*

1. *the S1 equilibria described in Lemma 1, in which the high type plays* $p_1 = 0$, $\Gamma_1 = \Gamma^*$, $p_2 = q_H$, $\Gamma_2 = C - \Gamma^*$.

2. *the set of S2 equilibria that satisfy Lemma 2 and Lemma 4, in which the high type play* $p_1 \leq \bar{q}$, $\gamma_1 \leq \gamma^*$, $p_2 = q_H$, $\gamma_2 = n - \gamma_1$ *and* $\pi_H > (n - \gamma^*)q_H$.

3. *the set of NS equilibria that satisfy Lemma 2 and Lemma 4, in which the high type play* $p_1 \leq \bar{q}$, $\gamma_1 \leq \gamma^*$, $p_2 \leq \bar{q}$, Γ_2 *and* $\pi_H > (n - \gamma^*)q_H$.

This follows directly from the Lemmas. While the Intuitive Criterion does not select a unique equilibrium in this game, we can draw some concrete conclusions from Theorem 1. No equilibrium features more than γ^* offers in sales round 1, so that the monopolist never visits more than a number of consumers equal to the domination number in any equilibrium that survives the application of the Intuitive Criterion. The lower the domination number γ^*, the more work the social network structure does to spread news of the firm's type, and so the lower the number of consumers who are targeted in round 1 as part of a signaling strategy to reveal type before round 2. This result extends previous work on fostering communication as a signal (Kennedy, 1994, Vettas, 1997) to a setting with explicit targeting that fosters communication.

The most profitable S2 equilibrium is the most profitable overall for the high type firm. In this equilibrium the two types of firm 'pool' on the minimum dominating set at the maximal pooled price, $\bar{q}$, in round 1, and the high type sells at the separated price q_H in round 2 to all remaining consumers; this can be supported, for example, by the following strategies and suitable beliefs:

$$\begin{aligned}
\sigma_{F,L} &: \{\bar{q}, q_H, \Gamma^*, C - \Gamma^*\} \\
\sigma_{F,H} &: \{\bar{q}, q_H, \Gamma^*, C - \Gamma^*\} \\
\sigma_{1,i} &: \{(b = 1 | p_1(i) = \bar{q}), (b = 0 | p_1(i) \neq \bar{q})\} \\
\sigma_{2,i} &: \{(b = 1 | p_2(i) \leq \hat{q}), (b = 0 | p_2(i) > \hat{q})\}
\end{aligned}$$

All consumers receive an expected payoff of zero. The payoff to each type of firm is as follows:

$$\pi_L = \gamma^* \bar{q} \tag{4}$$

$$\pi_H = \gamma^* \bar{q} + (n - \gamma^*) q_H \tag{5}$$

In the recurring double star example, this would involve the shaded consumers targeted in round 1 and purchasing at $p_1 = \bar{q}$, followed by all other consumers targeted in round 2 and purchasing at $p_2 = q_H$:

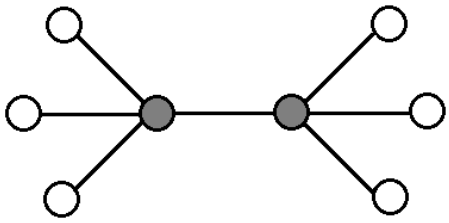

Figure 4: Minimum dominating set targeted in first round by both types in S2 equilibrium

As before, consumers not targeted in round 1 go on to hear from the targeted consumers the true quality of the firm; tomorrow good firms can offer these consumers the product at a price consistent with high quality, and low quality firms are driven out. A distinction with S1 is that now the communication from the early targets to the rest of the market is not simply confirming the inference all were able to make over the launch strategy, but is directly revealing information that the wider population could not directly infer. The nature of the set targeted in the first round in both the S1 and efficient S2 cases is such that it is the most parsimonious that covers the whole network with word-of-mouth; identifying this set requires considering the connectivity of sets of nodes taken together, rather than only considering individual measures of connectivity. This finding is therefore relevant to the question of whether campaigns informed by network structure should seek to identify and target consumers occupying a central network position, as discussed in Iyengar et al. (2011b).

This S2 equilibrium is clearly more profitable for the high type firm than the S1 equilibrium. The fundamental difference between the two classes of equilibria is, again, that in the between-round separation case the types are not separated in round 1, while in the pre-round 1 separation case the high type firm prices at a discount sufficient to separate before round 1. The high type thus earns positive profits in the pooled first round; it is these first-round profits that must be foregone in order to prove type in order to separate before round 1.

Theorem 1 demonstrates that the Intuitive Criterion clearly admits a large class of equilibria depending on the beliefs held by consumers. In particular, this refinement cannot put a *lower* bound on the size of the set of consumers to whom offers are made in round 1. In the extreme, if consumers believe that an out-of-equilibrium $\gamma_1 > 0$ must come from a low type, $\Gamma_1 = 0$ can be admitted in equilibrium, despite the firm's clear incentive to generate word-of-mouth, if the round 2 component of the equilibrium makes it more profitable than the fallback S1 equilibria. This extreme case could be 'blamed' on the extreme pessimism of those supporting beliefs. The following section explores an equilibrium refinement that will restrict further consumers' beliefs and be sufficient to select a unique equilibrium in this game.

4 Equilibria surviving passive conjectures

Consider again the "efficient" S2 equilibrium:

$$\begin{aligned}
\sigma_{F,L} &: \{\bar{q}, q_H, \Gamma^*, C - \Gamma^*\} \\
\sigma_{F,H} &: \{\bar{q}, q_H, \Gamma^*, C - \Gamma^*\} \\
\sigma_{1,i} &: \{(b = 1 | p_1(i) = \bar{q}), (b = 0 | p_1(i) \neq \bar{q})\} \\
\sigma_{2,i} &: \{(b = 1 | p_2(i) \leq q_H), (b = 0 | p_2(i) > q_H)\}
\end{aligned}$$

All consumers receive an expected payoff of zero. The payoff to each type of firm is as follows:

$$\pi_L = \gamma^* \bar{q} \tag{6}$$

$$\pi_H = \gamma^* \bar{q} + (n - \gamma^*) q_H \tag{7}$$

It is unambiguously true that this is preferred by the high type firm to the S1 equilibria, since the round 1 pooling is certainly preferred to incurring the cost of making the price discount to separate in round 1. But, since this pooling in round 1 benefits both high and low types, the Intuitive Criterion admits consumer beliefs that do not accept a deviation to this strategy.

For this 'pooled price' in round 1 to be acceptable to the targeted consumers, it must be that when consumers see an out of equilibrium action they place sufficiently high probability on it having come from a high type. One possibility would be to impose a **passive conjectures** property, as discussed in, for example, Rubinstein (1985) and Rasmusen (2001). It is defined here as follows:

Definition 4. ***Passive conjectures****: When consumers see an out of equilibrium action that would be profitable for either type of firm if it was accepted, they maintain their prior beliefs, which place probability α on the deviation coming from a high type and probability $1 - \alpha$ on the deviation coming from a low type, the true exogenous propensities of each type of firm.*

Passive conjectures can be motivated as, for example, the consumers perceiving out-of-equilibrium play as a mistake that is equally likely to be made by either type of firm. The key feature of passive conjectures is that when a consumer is offered the good under a price and visitation menu they did not anticipate, they are still willing to purchase the good if the unexpected price does not exceed mean product quality in the population of firms.

Theorem 2. *When consumers' beliefs satisfy passive conjectures, all equilibria in any network are efficient S2 equilibria.*

Proof. With passive conjectures, a unilateral deviation by the high type firm to the strategy specified in an efficient S2 equilibrium will result in all consumers who are offered a price in

round 1 under that strategy accepting the offer: the round 1 price of $\bar{q}$ is acceptable under the assumption that consumers place probability α on the unknown firm being of high type. Since efficient S2 equilibria are the most profitable for the high type firm, this unilateral deviation is always attractive to the high type firm. □

Under passive conjectures, consumers who are targeted in round 1 are willing to 'take a chance' on a product that they cannot infer to be certainly good. The high type firm then prefers to pool temporarily with the low type rather than to give a costly price discount to prove its type, and so prefers to play the strategy in the efficient S2 equilibrium than the immediately separating strategy of the S1 equilibrium. It is still the case that the targeted consumers are those whose word-of-mouth will blanket the network, but in this equilibrium high quality is discovered by targeted consumers who buy without knowing it with certainty, and so the market learns quality entirely through experience and word-of-mouth. We can view these equilibria as anti-signaling: no consumer infers the product's true quality from the firm's strategy before they purchase. Good products are 'discovered' by the market, as both high- and low-quality products are initially traded before low quality is driven out.

An important feature of the efficient S2 equilibrium strategy in relation to the S1 equilibrium strategy is that the firm earns revenue from first-round sales in the S2 equilibrium, where in S1 it had to burn money equal to first-round revenue in order to signal type. The spirit of the S2 strategy can be observed, for example, in the Iyengar et al. (2011b) study of contagion in the diffusion of a new pharmaceutical product. In that case, once individuals who were found by sociometric measures to be opinion leaders were identified, they received personal selling visits but, for reasons related to the nature of the drug, they did not receive free samples. We can observe a strategy of selling at a discount to select consumers in the case of Approach Software, who in the early 1990s launched their new database product by offering it first to a set of small company CEOs, who they identified as 'influential users', at a steep price discount, and then at full price as word-of-mouth spread (Griffin, 1995, chap. 8). As was the case with the S1 strategy, services that offer firms the ability to tailor discounts according to a consumer's position in the network of influence in a product category are also consistent with the S2 strategy, with the distinction that under an S1 strategy the product would be given away to such consumers, while under S2 the product would be sold at a discount. We can observe both giveaways and discounts offered by firms that use these services. Examples of discounts include the online fashion retailer Gilt Groupe using Klout scores to offer promotions that offer consumers a discount proportional to their influence in online social graphs (Gilt Groupe, Inc., 2012), and Stitch Fix, a novel online personal shopping service, similarly using the Klout Perks platform to offer a discount on a consumer's first order from the site (Klout Inc., 2014). In all of these examples, the key feature is that the consumers targeted are willing to take a chance on the product, able to infer that their influence among other potential consumers is such that if the product is good, the firm will benefit from the information they

spread, while if the product is bad, the firm will suffer. Whether a giveaway or a discount for the selected consumers is more appropriate may depend on the product's novelty and price; these contribute to a consumers' willingness to 'take a chance' on the product, captured in the model by the structure of consumer beliefs that support each class of separating equilibrium in the model.

Formally, in order to select these "efficient" S2 cases as the unique set of equilibria in the game, it must be the case that the high type firm can deviate from the strategy associated with a less profitable equilibrium—that is, to set a higher price or visit more consumers than 'expected'—and have this menu be accepted by round 1 consumers. The ability of the firm to forgo costly signaling in this way therefore depends on how consumers interpret a firm's launch strategy.

In the case in which the network has a unique minimum dominating set, Theorem 2 selects a unique equilibrium, and in the case in which there are multiple minimum dominating sets the equilibria are qualitatively unique in that they all feature the firm visiting some minimum dominating set in round 1. While under passive conjectures the qualitative outcome is independent of the precise network structure, the quantitative outcome in this unique equilibrium varies with network structure in the same way as in the S1 equilibria with costly up-front signaling.

Corollary 2. *In the unique (S2) equilibrium surviving the application of passive conjectures, the marginal value to the high type firm of adding a link to the network is either* 0 *or* $(q_H - \bar{q})$*, depending on the location of the link. The marginal value to the low type firm of adding a link is either* 0 *or* $-\bar{q}$*, depending on the location the link.*

The lower is the domination number γ^*, the fewer consumers need be targeted in order to saturate the network with word-of-mouth and the more can then be served later at the full high quality price. Thus adding links to a given network is weakly profitable for the high type firm and weakly reduces the payoff for the low type. However, given two arbitrary networks with the same number of of consumers, the network with a greater number of links will not necessarily be preferred by the high quality producer. For example, a circle network can have a higher average number of neighbors per consumer than a star network with one central consumer yet have a larger domination number.

5 Observability of the network to consumers

In this section we consider how the equilibrium outcomes in the game would change if the network structure was not observable to consumers. Alternative assumptions could be that consumers know who they are connected to but no-one beyond that, or that consumers do not know anything about the network, instead knowing only if they receive an offer and at what price. The key relevant insight when relaxing the observability assumption is that the results of the model imply that if the high type firm can discount enough to convince the set that will maximize word-of-mouth to buy, then there is no *extra* benefit to any other signaling strategy based on local promotions, even

though consumers can observe the network. That is, the high type firm's strategies in its preferred separating equilibria do not require consumers who are not targeted in the first round to infer anything about quality from the firm's global targeting behavior. With this is mind we can arrive at conditions under which the high type firm's strategies from the observable network case also apply to the unobservable network case.

In the S1 equilibrium, the high type firm made first round offers at zero price to a minimum dominating set of the network graph. This strategy could never be profitable for the low type firm, and so all consumers can infer that the firm is of high type: since consumers can observe the network structure, they know that when these first round offers are accepted, word of mouth on quality will cover the whole network, leaving the low type frozen out. If consumers cannot observe the whole graph, they cannot make this inference. However, under a free disposal assumption—equivalently the low product having a quality $q_L = 0$—then the high type's strategy in S1 equilibrium is preserved even when consumers observe nothing about the network. The reason for this is twofold. First, with $q_L = 0$ a price of zero is always weakly acceptable in equilibrium to a consumer who receives it, and so the high type can make offers to any such set it chooses and have them be accepted. Second, as we saw in Lemma 1, the high type's preferred strategy among the S1 class did not involve any consumers being asked to purchase the good who did not either receive a zero price offer or learn true quality by word of mouth. Since no consumer who did not receive a zero price offer is asked to make inference from the targeting strategy, as long as the zero price is weakly acceptable we can arbitrarily degrade consumers' knowledge of the network structure without destroying the high type's efficient S1 strategy.[10] The strategies that a high type firm can play to separate that would not survive under an unobservable network are therefore those which were already ruled out by Lemma 1 as being inferior for the high type firm than its preferred S1 strategy.

A caveat to this point is that when the network is observable to consumers we can be quite flexible in the meaning of the model's 'price discount'. As discussed above, any money burning action that costs at least as much as first-period revenue is sufficient to preclude profitable imitation by a low type as long as the number and location of offers in the first round is observable to consumers. If consumers do not know the location of offers, it must be that each consumer who receives a first round offer is weakly willing to buy independently of the firm's strategy in distant network areas, and so the interpretation of the money burning action as a price discount must be taken more literally.

The conclusion of Lemma 2, which placed a lower bound on high type profit in the game as a whole under the observable network assumption, will therefore continue to hold with a network unobservable to consumers under this extra structure. The high type will still be able to select

[10]We may also note that if we maintain $q_L < 0$, so that a low quality product has negative value to a consumer, the analog of the S1 equilibrium with a network unobservable to consumers would involve a *negative* price sufficient to make purchase weakly acceptable to each targeted consumer, although naturally the payoff to the high type would be lower. This strategy would be equivalent to a firm paying consumers to 'test' a new product.

the S1 strategy and earn this payoff, and so strategies from the pooling (NS) and pooling-then-separating (S2) classes that do not offer at least much will not be observed. High type strategies in this case will then still be consistent with those that survived the Intuitive Criterion under Theorem 1. What cannot be ruled out is that the low type firm may earn a better payoff than in the model's S1 equilibrium. When the network is unobservable to consumers, the S1 strategy can be available to the high type and cannot be profitably imitated by the low type, but the same strategy by the high type now does not necessarily rule out low type strategies that are not part of the set of equilibria of Theorem 1. This is because when the network is unobservable to consumers, when consumers see the firm take some action they cannot in general tell whether it could have been part of a strategy preferred by the high type firm to the S1 equilibrium or not. Thus under the unobservable network assumption there may be more strategies available to the low type that the consumers cannot rule out as having come from the high type.

The strategy played by the high type in the efficient S2 equilibrium is also robust to relaxing the assumption that consumers know the network structure, as long as we retain the passive conjectures restriction on consumer beliefs. The reasoning is similar to the survival of the S1 strategy: as long as the consumers who receive offers in the first round hold beliefs such that they are willing to accept, the rest of the equilibrium strategy follows identically since no consumers are called on to infer type in the second round rather than learn it by word-of-mouth. When consumers form beliefs by passive conjectures, when they observe a zero probability (out of equilibrium) strategy, they maintain their prior, which in this case is the population distribution of high and low types. As in the observable network case, since the efficient S2 equilibrium strategy yields the highest payoff to the high type firm among all equilibria, we can retain for the high type the conclusion of Theorem 2 that if consumers' beliefs satisfy the passive conjectures restriction, we have a qualitatively unique prediction for the high type's strategy.

While the nature of the high type firm's strategy can therefore be robust to relaxing the assumption that consumers observe the nature of the network, there may be some settings in which this assumption can be appropriate. Relatively small and close-knit groups are more likely to have more well understood networks of relationships, for example within an organization, where the organizational form is well-known and correlates with social interactions, or within a school, where the composition of cliques and friendship groups is largely common knowledge. Although survey methods to construct the shape of networks typically ask respondents about their own connections rather than the overall shape of the network, the fact that they have proven useful to discover empirically relevant aspects of networks of relationships (for example Tsai and Ghoshal, 1998 in the case of intrafirm networks and Godes and Ofek, 2004 in the case of a population of schoolchildren) suggests that in small-scale settings network members would be capable of holding a picture of the whole structure. In applications in which the target 'consumers' are firms, the size of the inter-firm network—capturing such aspects as formal contracts, relational contracts, proximity in industrial clusters, and co-working (see Grandori and Soda, 1995)—can also be relatively small

and be determined by observable factors.

In some cases characteristics of the product may also mean that the assumption that consumers know the network structure is more appropriate, even if the number of consumers is quite large. If the product is specialized and so of interest to a niche market, the spaces in which potential consumers interact are likely to be more well-defined and fewer in number than for general product categories. This is particularly the case online, where discussion forums and communities around niche interests are abundant. A key question in this context is whether testimony in online communities is trustworthy (see, for example, Mayzlin et al., 2012), an issue we discuss below in Section 6. If this is the case, then important aspect of the shape of the network is then simple enough to be well-understood by consumers: information shared by one consumer who belongs to the online community can in principle reach the whole community.

Beyond these possibilities, there may be special cases in which consumers may not know the whole structure but have sufficiently good information about its shape as to draw inference from the firm's strategy. For example, in industries with a well-defined set of reviewers or critics, consumers in the large can infer something about quality from a firm's decision on whether or not to release products for review before they are available to the public. Even though the social graph among consumers is not necessarily well known, the set of critical outlets whose audiences cover the relevant market may be more well defined for consumers. It is common practice in the video game industry to set 'review embargo dates' to control when those individuals who are the first to receive the product can write about it, and it is commonly believed that using this policy can be telling: as one industry expert puts it, "[i]f you know your game is going to get thrashed in reviews, you're definitely going to set a launch-day embargo, or you may not send out early review code at all" (Alan, 2012). The same inference is routinely made for movies that are not screened for critics. A typical example from the movie review aggregator Rotten Tomatoes reads "[w]e'd love to tell you more about this one, but it doesn't screen for critics until later in the week, which is never a good sign" (Ryan, 2013).[11] Similarly, in the cases of giveaways to celebrities or other visible tastemakers, while consumers may not know the whole network of relationships, they can nonetheless draw similar implications if they perceive that the targeted consumers will be widely observable to the population at large. One way to view these examples is as cases in which consumers do not observe the full network but know enough about its structure to infer something from the firm's behavior.

[11] A caveat to this point is that it will apply only to the population of 'sophisticated' consumers. Brown et al. (2012) demonstrate that, controlling for measures of quality, a revenue premium for movies not screened for critics, which they interpret as evidence for limited rationality among a subset of consumers.

6 Information sharing assumptions and credibility

An assumption of the model is that connected consumers share information automatically, honestly, and precisely. This means that the high quality firm gets a substantial benefit from purchases by consumers in the launch stage. If consumers were less willing or reliable in sharing information, then if the firm targets the minimum dominating set as in the efficient S1 and S2 equilibria, we should expect fewer consumers to learn quality by word-of-mouth tomorrow. The implication in the signaling framework is that now some consumers will not learn quality by word-of-mouth and so a larger price discount than before will be necessary if those consumers are to be able to infer quality from the firm's strategy. This is analogous to the situation in the inefficient S1 equilibria that were ruled out under Lemma 1. Therefore, with imperfect information sharing, to achieve the same separation under the Intuitive Criterion as in the surviving S1 equilibria, the firm would have to increase the number of consumers targeted in the first round or offer a larger price discount to those targeted.

Under the passive conjectures refinement, consumer beliefs are sufficiently optimistic that the high type can retain the first round strategy from the efficient S2 equilibrium. However, again the high type is hurt by imperfect information sharing, this time because fewer consumers will have learned the firm's type by the second round. It is therefore again true that the firm may prefer to target more consumers in the first round, since redundancy in the set of neighbors to targeted consumers can now be valuable, increasing the chance that those neighbors will be reached by word-of-mouth, rather than strictly costly as before.

Another implication both of the setting that we consider and the assumption of objective information sharing is that the high quality firm has a strong incentive to try to create a single 'consumer' (or a small set of consumers) who is connected to all others, giving a minimum dominating set of cardinality one. With honest and precise communication, this gives a very cheap path to signaling quality for the high type firm, since they need only to offer a price discount or money burning action sufficient to convince one consumer to buy, before selling to all other consumers at the full high quality price. The problem with this argument is that the created node would be an agent of the firm, and so the remaining consumers may question the credibility of the single first round target. This individual would be corruptible, subject to an incentive to give false quality reports in exchange for side payments from a low type firm.

In general, the credibility problem when using an agent can be mitigated by methods of commitment, such as the agent's compensation contract being public knowledge (Vickers, 1985, Fershtman and Judd, 1987, Fershtman et al., 1991) or the contract being private knowledge but information on quality being private to the agent and unknown to the principal (Caillaud and Hermalin, 1993). In the present context, these credibility and agency concerns and the value to the firm of better information sharing by consumers can be related justifications for a firm to rely on existing network and communication structures as commitment devices. In particular, two interpretations of

the nature of the first round targets in the model are as a set of professional or hobbyist critics, playing a repeated game with their audiences or followers, or a set of regular consumers, relaying information to their social or professional contacts. In the first case, the repeated nature of the interaction between the critic who provides information on quality and her audience who receives the information gives rise to reputation concerns which could help to mitigate corruptibility and enhance credibility. In the second case, the incentive for the targeted individuals to maintain their pre-existing relationships plausibly will work in the same direction.

Whether one or the other of these interpretations is the right way to think about the nodes targeted in the first round will depend on the product and the network. For product classes that are familiar, frequently consumed, or have a large potential audience, it may be viable to formalize the role played by the 'consumers' targeted in the first round of our model into the institution of a criticism and review industry. For product classes that are novel, infrequently consumed, or have a small potential audience, it is likely to be difficult to create or sustain a credible layer of well-connected individuals functioning as critics, and so targeting regular consumers, perhaps selected by network analysis or discovery of the social graph may be a more promising approach to generate trusted inference and word-of-mouth. In both cases the model's assumptions of automatic and honest communication can be justified in part by the incentives of the targeted consumers. In this way, exploiting pre-existing network structures could function in the same way as a commitment device, to help the firm to ameliorate credibility and agency concerns. Determining the best approach for the firm may therefore involve a trade off between creating well-connected consumers and designing commitment mechanisms to enhance their credibility versus accepting a larger set of first round targets but with less need to construct such mechanisms.

7 Concluding comments

This paper has studied a quality signaling game in which the firm can use publicly observable localized promotions in the consumers' communication network. In keeping with previous results on signaling games with communication, the firm restricts early activity to foster communication. Here early activity is restricted more when the network can be more easily blanketed with word-of-mouth. In separating equilibria, the firm targets an idiosyncratic set of dispersed consumers in order to use communication to spread word about its quality while leaving as many consumers as possible to be served tomorrow after this information spreads.

Identifying this set requires considering which sets of nodes are collectively the best-connected to cover the whole network with word-of-mouth, rather than simply considering measures of connectivity for individual nodes. Which nodes belong to such a set is an empirical question: depending on the network structure, the set may be composed of relatively high-degree nodes or also include some low-degree nodes. The distinction between individual measures and collective measures is that if the nodes who individually have the most connections are clustered together—so that nodes

with a large number of connections are likely to be connected to other nodes with a large number of connections—then considering only individual connections risks 'oversampling' clusters of highly connected nodes, creating redundancy in the resulting word-of-mouth (Iyengar et al., 2011a demonstrate an example of such redundancy in the location of opinion leaders in a network of San Francisco physicians). Thus identifying the individually best-connected consumers is not necessarily the most valuable service that a targeting platform can offer; more important is identifying a set of consumers who will cover all areas of the network, both dense and sparse. This result was especially stark in our two-period model, but readily generalizes upwards: the firm will prefer to target the smallest set that blankets the network in a 'timely' fashion that will depend on the firm's degree of patience in the launch stage.

To convince the first targeted consumers to buy requires an associated money-burning action, whose magnitude is smaller when the network can be covered with word-of-mouth by initiating it at fewer locations. In this way the publicness of communication networks and the more connections in them are valuable for a seller of a high quality product, and also socially, since the amount of costly signaling required to 'discover' the high quality products is smaller. This represents one possible justification for businesses and policymakers to facilitate the development of these technologies.

Separating equilibria exist in which the high quality firm is separated immediately, taking a large money-burning action along with its targeting strategy in order to demonstrate its quality surely to those targeted early. However, there also exist separating equilibria in which high and low quality products at first coexist in a pooling stage before they are separated by word-of-mouth on their quality. For the sender of information the second class is preferable, but it requires early adopters to act 'on faith'. Understanding how consumers will respond to the the magnitude of the money-burning action is therefore an important consideration in the targeted launch strategy.

We can view the early-targeted consumers as the analog in this signaling model of opinion leaders and key influencers, which practical 'network analysis' seeks to identify in brand-building through word-of-mouth. In this model, however, they are important due to their idiosyncratic network position and not due to any other heterogeneity. In a richer setting characteristics like experience, credibility, trustworthiness and charisma will likely also be important. The signaling role of network seeding does suggest, however, that aiming to foster a broadly dispersed set of early adopters is more valuable to informers than simply generating the greatest possible volume of 'buzz'.

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A Proofs

A.1 Lemma 1

Proof. First we will demonstrate that at most one high type payoff value is realized in S1 equilibria that survive the application of the Intuitive Criterion.

We require $\hat{p}$ such that a low-type firm imitating the high-type strategy completely would receive zero profit in expectation. The potential credible imitator would make γ_1 sales at $\hat{p}$ in round 1 and $(n - \gamma_1 - \omega(\Gamma_1))$ sales at q_H in round 2 (since neighbors to consumers in Γ_1 observe the true type, they will not buy from the imitator in round 2). $\hat{p}$ must thus satisfy:

$$\begin{aligned} \pi_{L,imitate} &= \gamma_1 \hat{p} + (n - \gamma_1 - \omega(\Gamma_1))q_H && (8) \\ &\leq 0 && (9) \\ \Rightarrow \hat{p} &\leq -\frac{(n - \gamma_1 - \omega(\Gamma_1))q_H}{\gamma_1} && (10) \end{aligned}$$

The profit to the high type is thus:

$$\pi_H = \gamma_1 \hat{p} + (n - \gamma_1)q_H \quad (11)$$

All S1 equilibria that do not maximize this profit with respect to visitation γ_1 and price $\hat{p}$ fail to survive the application of the Intuitive Criterion. By the Intuitive Criterion consumers will place zero probability on a deviation from any such equilibrium to the profit-maximizing S1 equilibrium coming from the low type firm; the deviation is profitable for the high type and will be made.

Next we will go on to characterize this unique level of profit made by the high type firm in S1 equilibrium. Consider the S1 equilibrium in which $\Gamma_1 = \Gamma^*$. In that case, $\omega(\Gamma_1)$, the number of neighbors to consumers in Γ_1, is equal to $n - \gamma^*$, since by the definition of Γ^* all remaining consumers are neighbors of those in Γ^*. The maximal price in that case is:

$$\begin{aligned} \hat{p} &= -\frac{(n - \gamma^* - \omega(\Gamma_1))q_H}{\gamma^*} && (12) \\ \hat{p} &= -\frac{(n - \gamma^* - (n - \gamma^*))q_H}{\gamma^*} && (13) \\ \hat{p} &= 0 && (14) \end{aligned}$$

The profit to the high type is thus:

$$\pi_{H,1} = (n - \gamma^*)q_H \quad (15)$$

Now consider any other S1 equilibrium, with generic Γ_1. This has a maximal price:

$$\hat{p} = -\frac{(n - \gamma_1 - \omega(\Gamma_1))q_H}{\gamma_1} \tag{16}$$

Which yields profit to the high type:

$$\pi_{H,2} = -(n - \gamma_1 - \omega(\Gamma_1))q_H + (n - \gamma_1)q_H \tag{17}$$

Now comparing the two:

$$\pi_{H,2} > \pi_{H,1} \tag{18}$$

$$\text{if } \omega(\Gamma_1) > n - \gamma^* \tag{19}$$

But $\omega(\Gamma_1) \leq n - \gamma^*$ if $\gamma_1 \geq \gamma^*$ and $\omega(\Gamma_1) < n - \gamma^*$ if $\gamma_1 < \gamma^*$, so it is never true that $\pi_{H,2} > \pi_{H,1}$. The most profitable S1 equilibrium thus features $\Gamma_1 = \Gamma^*$ and has profit given by:

$$\pi_H = (n - \gamma^*)q_H \tag{20}$$

□

A.2 Lemma 2

Proof. The S1 strategy $\sigma_{F,H} : \{0, q_H, \Gamma^*, C - \Gamma^*\}$ is profitable for the high type but not profitable for the low type firm, so when consumers see this strategy they must place zero probability on it coming from the low type (by the Intuitive Criterion). The high type earns payoff $\delta(n - \gamma^*)q_H$ by playing this strategy, so will prefer to deviate to it from any other strategy that yields a lower equilibrium payoff. □

A.3 Lemma 3

Proof. Assume not, so that there exists an equilibrium with some Γ_1 and $\Gamma_2 = \emptyset$. Consider three cases:

1. Take $\Gamma_1 = C$. Consider a deviation to $\Gamma_1 = C - i$, $\Gamma_2 = i$ by the high type firm. Consumer i will buy since she will learn that quality is q_H; this is profitable for the high type firm since $q_H > \bar{q}$, but not profitable for the low type firm, which would lose consumer i.

2. Take $\Gamma_1 \subset C$, $\Gamma_1 \neq \emptyset$. Consumers in $\Omega(\Gamma_1)$ learn that quality is q_H. A deviation to $\Gamma_2 = \Omega(\Gamma_1)$, $p_2 = q_H$ is profitable for the high type firm, but (weakly) not profitable for the low type firm since those consumers learn true quality before stage 2 and so will not buy from the low type.

3. Take $\Gamma_1 = \emptyset$. Consider a deviation to $p_1 = 0$, $\Gamma_1 \neq \emptyset$, $\Gamma_2 = \Omega(\Gamma_1)$, $p_2 = q_H$. This is profitable for the high type firm, but (weakly) not profitable for the low type firm, which makes no profit in stage 1 and no sales in stage 2.

In each case there is a deviation to $\Gamma_2 \neq \emptyset$ that is profitable for the high type but not the low type. By the Intuitive Criterion, consumers must place zero probability on that deviation coming from the low type and so will accept the deviation's offers; this is profitable for the high type. $\Gamma_2 = \emptyset$ could not have been part of an equilibrium. □

A.4 Lemma 4

Proof. Assume not, so that $\gamma_1 > \gamma^*$ in an equilibrium that survives the Intuitive Criterion. Consider two cases:

1. If consumers in Γ_1 choose not to buy in round 1 or if $p_1 < 0$, the high type firm cannot realize a higher payoff than in the equilibria of Lemma 1. By that result, the high type can profitably deviate to an S1 equilibrium, and since the low type cannot profit from such a deviation consumers place zero probability on the deviant being of low type.

2. If consumers in Γ_1 choose to buy in round 1, the high type firm can deviate to $\Gamma_1 = \Gamma^*$, $\Gamma_2 = C - \Gamma^*$, $p_2 = q_H$ and p_1 unchanged. This deviation is profitable for the high type firm since $q_H > \bar{q}$, but is not profitable for the low type since it reduces sales in round 1 and does not increase sales in round 2, since word-of-mouth reveals type to all in $C - \Gamma^*$.

$\gamma_1 > \gamma^*$ could not have been part of an equilibrium surviving the Intuitive Criterion. □